RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)

THIRD YEAR
B.A./B.SC. FIFTH SEMESTER (July – December) 2014
Mid-Semester Examination, September 2014

Date : 15/09/2014 **MATHEMATICS (Honours)**
Time : 2 pm – 4 pm Paper : V Full Marks : 50

[Use a separate answer book for each group]

Group – A

(*Answer* ***Question No. 1*** *and* ***any four*** *from the rest*)

1. a) Show that the group $(\mathbb{Q},+)$ can't be expressed as an internal direct product of two nontrivial subgroups. [3]
   b) Can the cyclic group $\mathbb{Z}_{12}$ be expressed as an internal direct product of two proper subgroups? Justify your answer. [3]
   c) Show that $\mathbb{Z}\times\mathbb{Z}$ is not cyclic. [3]
2. Does there exist a nontrivial group homomorphism from $(\mathbb{R},+)$ to $(\mathbb{Z},+)$? Justify. [4]
3. Let G be a group of order 8 and $x\in G$. If o(x) = 4, prove that $x^2\in Z(G)$. [4]
4. Prove that the only proper subgroup of $(\mathbb{R}^*,\bullet)$ of finite index is $\mathbb{R}^+=\{x\in\mathbb{R}:x>0\}$, here $\mathbb{R}^*=\mathbb{R}-\{0\}$. [4]
5. Give an example of an infinite group G such that for each $n\in\mathbb{N}$, $\exists x\in G$ with o(x) = n. [4]
6. Let G be an abelian group of order 8. Prove that $\phi:G\to G$ defined by $\phi(x)=x^3\ \forall x\in G$ is an isomorphism. [4]
7. Let G be a group and $a\in G$. Define $f:G\to G$ by $f(x)=axa^{-1}\ \forall x\in G$. Prove that f is an automorphism. [4]

Group – B

8. Answer **any three** questions : [3×5]
   a) Define forward difference of a function f. If $\phi_r(x)=(x-x_0)(x-x_1)...(x-x_r)$ where $x_r=x_0+rh$, $r=0,1,2,...,n$, calculate $\Delta^k\phi_r(x)$. Hence or otherwise, obtain Newtons forward interpolation formula without the error term. [2+3]
   b) Establish Newton – Cotes' formula for numerical integration. (Error in not necessary).
   Deduce Simpson's one third rule from the above formula. State the geometrical interpretation of the one third rule. [3+2]
   c) Define numerical differentiation. Establish the numerical differentiation formula based on Lagrange's interpolation. [1+4]
   d) Show that the error in approximating f(x) by an interpolating polynomial is $(x-x_0)(x-x_1)...(x-x_n)\dfrac{f^{n+1}(\xi)}{\underline{|n+1}}$, where ξ lies between the smallest and the greatest of the numbers $x_0, x_1, ..., x_n$ [5]

e) Using Euler's modified method, obtain the solution of the differential equation : $\frac{dy}{dx} = x + \sqrt{y}$, $y(0) = 1$ for the range $0 \le x \le 0{\cdot}4$ in steps of length $0{\cdot}2$. [5]

9. Answer **<u>any two</u>** questions : [2×5]

a) Let $f:[a,b] \to \mathbb{R}$ be integrable on [a,b] & $F(x) = \int_a^x f(t)dt$, $x \in [a,b]$. Prove that—

i) F is continuous on [a,b]

ii) F is of bounded variation on [a,b] [2½+2½]

b) Let $f:[a,b] \to \mathbb{R}$ be bounded on [a,b]. If f has a finite number of points of discontinuties in [a,b] then show that f is integrable on [a,b]. [5]

c) Let $f \in C[0,1]$ and $\int_0^1 f(x)dx = 0 = \int_0^1 xf(x)dx$. Then prove that there exist two distinct point $a < b$ in [0,1] such that f(a) = f(b) = 0. [5]

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